MATHS

Year 9
Answers

Lesson 6

Topic: Linear and non-linear relationships

Introducing gradients

Lesson concepts

- Functions — Tables and graphs
- Proportional reasoning — Rates
- Proportional reasoning — Direct proportion

Lesson notes

Students will explore the concept of slope.

Lesson answers

1. a. Slope is a measure and/or description of the steepness or flatness of an incline or decline.

   b. For example: Something that is very steep has a lot of vertical change over very little horizontal change.

   c. Steep or flat, sharp or gentle, rising or falling.

2. Students watch the **Video — Introducing slope**.

3. Plot 3 is a positive slope, as it goes up 1 unit for every unit it goes right and crosses the y-axis at 0.

   Plot 4 is a negative slope, as it goes down 2 units for every unit it goes right and crosses the y-axis at 11.

4. For example:

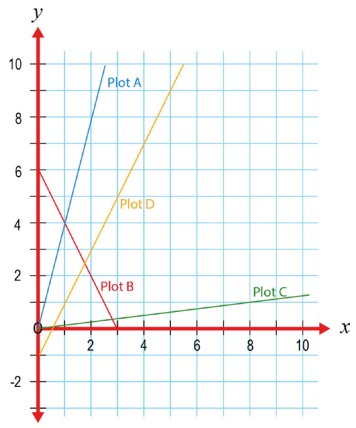
   Plot 2 (black) is positive. It goes up 2 units for every unit it goes right. It passes through the y-axis at 2.

   Plot 3 (blue) is negative. It goes down 3 units for every unit it goes right. It passes through the y-axis at 6.

   Plot 4 (purple) is positive. It goes up half a unit for every unit it goes right. It passes through the y-axis at 0.

   Plot 5 (green) is negative. It goes down half a unit for every unit it goes right. It passes through the y-axis at 8.

5. **Sheet 4** answers:



Plots A, C and D have a positive slope.

6. Students open **Sheet 5 — Tell a story**.

7. **Sheet 5** answers
   For example:

   Aaron, Beth, Cai and Diesel all left school at the same time. Diesel ran quickly for 5 seconds before realising he had forgotten his footy boots and headed back to school. Cai was in no hurry to get home, so walked slowly when she left. Aaron and Beth rushed from the school and had a race. Aaron ran faster but stopped after 10 seconds. Beth ran past him 1 second after he stopped.

Lesson 7

Topic: Linear and non-linear relationships

Calculating gradients 1

Lesson concepts

- Functions — Tables and graphs
- Proportional reasoning — Linear models
- Proportional reasoning — Direct proportion

Lesson notes

Students will:

- revise describing slopes
- explore the rule for calculating gradient.

Lesson answers

1. Sample response:
   Line 1 (black) is positive. It goes up 1 unit for every unit it goes right. It passes through the y-axis at 0.

   Line 2 (blue) is negative. It goes down 4 units for every unit it goes right. It passes through the y-axis at 6.

Line 3 (purple) is positive. It goes up 3 units for every unit it goes right. It passes through the y-axis at 0.

Line 4 (red) is positive. It goes up half a unit for every unit it goes right. It passes through the y-axis at 6.

2. Students watch the **Video — Gradient**.

3. a. Graph A has a gradient of 1. The rise = 1 and the run = 1

   b. Graph B has a gradient of 2. The rise = 2 and the run = 1

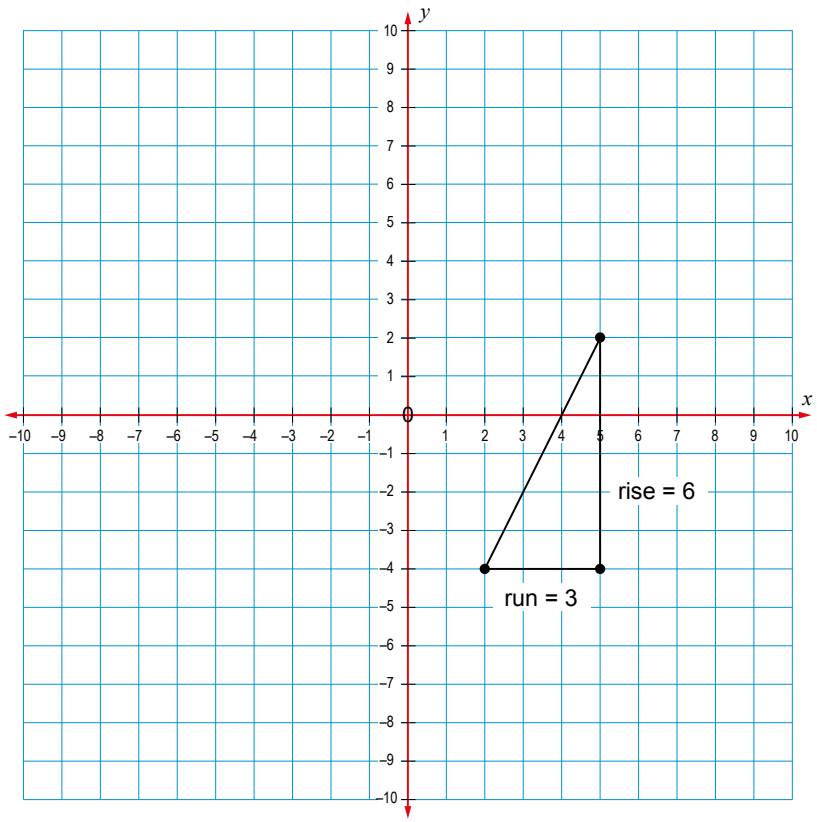
   c. Graph C has a gradient of 3. The rise = 3 and the run = 1

   d. What do you notice about the run for each graph? The run stays the same, that is, 1.

   e. How does the gradient change as the rise increases?
   For example: As the rise increases, so does the gradient.

   f. What type of line will you get if the rise = 0? (Try drawing the line on the Cartesian plane to help, if necessary). The line will be horizontal (flat).

4. a–f.



   g. gradient = $\frac{6}{3} = 2$

5. a. gradient = $\frac{4}{2} = 2$

   b. gradient = $\frac{1}{2}$

   c. gradient = $\frac{-2}{2} = -1$

6. a. Line 1 (black)

$$m = \frac{\text{rise}}{\text{run}} = \frac{1}{1} = 1$$

b. Line 2 (blue)

$$m = \frac{\text{rise}}{\text{run}} = \frac{-4}{1} = -4$$

c. Line 3 (purple)

$$m = \frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$$

d. Line 4 (red)

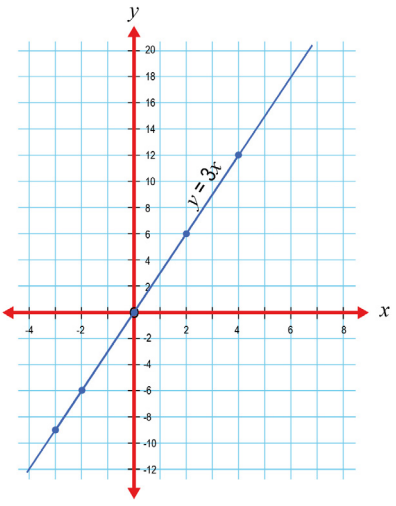
$$m = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$$

7. Students watch the **Video — Linear functions**.

8. **Sheet 6** answers:

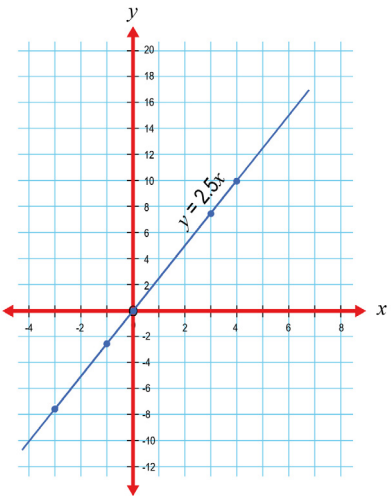
$y = 3x$

x	–3	–2	0	2	4
y	–9	–6	0	6	12



9. $y = 2.5x$

x	–3	–1	0	3	4
y	–7.5	–2.5	0	7.5	10



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10. a. 2.5

    b. (0, 0)

    c. 2.5

    d. It would become steeper. It would be twice as steep as the original line because 5 is twice 2.5.

    e. Measure the rise and run between two points on the graph and divide the rise by the run to get m.

Lesson 8

Topic: Linear and non-linear relationships

Calculating gradients 2

Lesson concepts

- Functions — Tables and graphs
- Proportional reasoning — Linear models
- Proportional reasoning — Direct proportion

Lesson notes

Students will reinforce their understanding of calculating gradients.

Lesson answers

1. m = 1, corresponds to the pink graph

   m = –2, corresponds to the blue graph

   m = 4, corresponds to the red graph

   m = 0, corresponds to the orange graph

2. Explore the link between gradient, y-intercept and features on a linear graph (straight line shown). Work through the following steps and answer the questions.

   a. Gradient = 2

   b. y-intercept = 3

   c. For example: The gradient is the coefficient of x (the number in front of the x variable) and the y-intercept is the constant added on at the end of the equation.

   d. Gradient = –2 and y-intercept = 7

      For example: The gradient is the coefficient of x and the y-intercept is the constant added on at the end of the equation.

e. Gradient = 3 and y-intercept = –6

For example: the gradient is the coefficient of x and the y-intercept is the constant added on at the end of the equation.

3. Students watch the **Video — Understanding the c value in a linear equation**.

4. Line 2

$m = \frac{\text{rise}}{\text{run}} = \frac{-1}{1} = -1$

$c = 5$

$y = -1x + 5$

$y = -x + 5$

Line 3

$m = \frac{\text{rise}}{\text{run}} = \frac{1}{1} = 1$

$c = 0$

$y = x$

Line 4

$m = \frac{\text{rise}}{\text{run}} = \frac{-1}{2} = -0.5$

$c = 8$

$y = -0.5x + 8$

5. a. Positive, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{3 - 2} = 3$

b. Positive, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{4 - 1} = \frac{1}{3}$

c. Negative, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{4 - 1} = -1$

6. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{-2 - (-2)} = \frac{4}{0}$ = undefined

This plot is rising 4 units while running 0 units. Therefore the plot is vertical, the gradient is undefined.

7. **Sheet 7 — How steep is it?** (attached).

8. a. $m = \frac{\text{rise}}{\text{run}} = \frac{20}{20} = 1$; The gradient of this staircase is 1.

b. $m = \frac{\text{rise}}{\text{run}} = \frac{20}{40} = 0.5$; The gradient of this staircase is 0.5. The stairs will be flatter.

c. $m = \frac{\text{rise}}{\text{run}} = \frac{1}{5} = 0.2$; No, because the gradient of this ramp is double the legal standard.

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d. $m = \frac{\text{rise}}{\text{run}}$, $0.1 = \frac{1}{x}$

$x = \frac{1}{0.1} = 10$

Therefore, the ramp would need a run of 10 m if it has a 1 m rise.

Lesson 9

Topic: Linear and non-linear relationships

Calculating the distance between two points 1

Lesson concepts

- Functions — Tables and graphs
- Representations — Linear models
- Length — Relationships
- Real numbers — Pythagoras and trigonometry

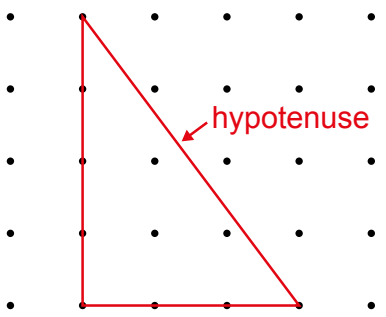
Lesson notes

Students will use Pythagoras' theorem to calculate the hypotenuse.

Lesson answers

1. The hypotenuse is:
   Side c in triangle one
   Side e in triangle two
   Side g in triangle three.

2. a. and b.



c. About 6 cm

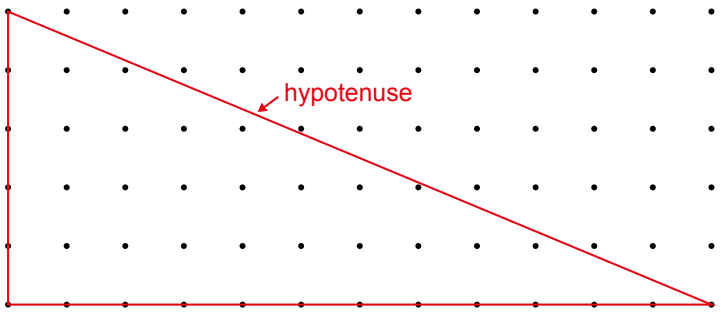
d. 5 cm

e. For example: It was too long.

f. For example: The hypotenuse is longer than the base.

g. For example: The hypotenuse is longer than the perpendicular height.

3. a. and b.



c. About 14 cm

d. 13 cm

e. For example: It was too long.

f. For example: The hypotenuse is longer than the base.

g. For example: The hypotenuse is longer than the perpendicular height.

4. Students watch the **Video — Pythagoras' theorem**.

5. a. $a^2 + b^2 = c^2$

$3^2 + 3^2 = c^2$

$9 + 9 = c^2$

$18 = c^2$

$\sqrt{18} = c$

$c \approx 4.24$ units

b. $a^2 + b^2 = c^2$

$5^2 + 5^2 = c^2$

$25 + 25 = c^2$

$50 = c^2$

$\sqrt{50} = c$

$c \approx 7.07$ units

6. a. Hypotenuse = 5 units

c. Hypotenuse = 10 units

d. Hypotenuse ≈ 8.6 units

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Lesson 10

Topic: Linear and non-linear relationships

Calculating the distance between two points 2

Lesson concepts

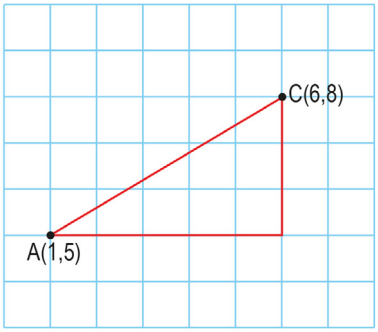
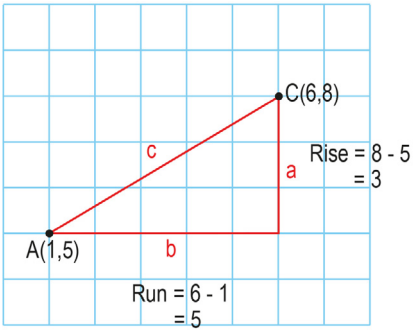
- Representations — Linear models
- Functions — Graphs
- Length — Relationships
- Real numbers — Pythagoras

Lesson notes

Students will calculate the distance between two points.

Lesson answers

1. a.

Think	Write
Mark the two points on the graph. May help to join the points with a line.	
Find the distance of the rise and the run. Label the sides of the triangle produced a, b and c. Recognise that the three lines form a right-angled triangle. Use Pythagoras' theorem to find the length of the hypotenuse.	
Recall Pythagoras' theorem, writing it with the hypotenuse as the subject of the formula.	$a^2 + b^2 = c^2$

Think	Write
Substitute the values for a and b into the formula. Solve the problem. Note the scale, which is 1 unit is equal to 1 cm.	$3^2 + 5^2 = c^2$ $9 + 25 =$ $34 =$ $\sqrt{34} = c$ $c \approx 5.83$ cm
Write a statement to communicate the final solution.	The distance of the line is approx 5.83 cm.
Use the following formula to calculate the gradient. Working from left to right, point A is (x^1, y^1) and the point C is (x^2, y^2). $m = \frac{\text{rise}}{\text{run}} = \frac{y^2 - y^1}{x^2 - x^1}$	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{8 - 5}{6 - 1}$ $m = \frac{3}{5}$ or 0.6

B (2, 3); D (9, 12)
Rise = 9; Run = 7

$a^2 + b^2 = c^2$
$7^2 + 9^2 =$
$49 + 81 =$
$130 =$
$\sqrt{130} = c$
$c \approx 11.4$ cm

The distance of the line is approx 11.4 cm.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{12 - 3}{9 - 2}$

$m \approx 1.29$

2. a. (4, 2) to (10, 6)
Rise = 4; Run = 6

$a^2 + b^2 = c^2$
$4^2 + 6^2 =$
$16 + 36 =$
$52 =$
$\sqrt{52} = c$
$c \approx 7.21$ cm

b. (2, 14) to (14, 2)
Rise = –12; Run = 12

$a^2 + b^2 = c^2$
$12^2 + (-12)^2 = c^2$
$144 + 144 = c^2$
$288 = c^2$
$\sqrt{288} = c$
$c \approx 16.97$ cm

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c. (9, 5) to (15, 11)
Rise = 6; Run = 6

$a^2 + b^2 = c^2$
$6^2 + 6^2 =$
$36 + 36 =$
$72 =$
$\sqrt{72} = c$
$c \approx 8.49$ cm

3. a. With coordinates of (3, 4) and (0, 0), the short sides of the triangles will be 3 and 4.

$a^2 + b^2 = c^2$
$3^2 + 4^2 =$
$9 + 16 =$
$25 =$
$\sqrt{25} = c$
$c = 5$ units

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0 - 4}{0 - 3}$$

$m \approx 1.3$

b. With coordinates of (0, 0) and (2, 3) the short sides of the triangles will be 2 and 3.

$a^2 + b^2 = c^2$
$2^2 + 3^2 =$
$4 + 9 =$
$13 =$
$\sqrt{13} = c$
$c \approx 3.61$ units

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0 - 3}{0 - 2}$$

$m \approx 1.5$

c. With coordinates of (–4, 3) and (0,– 3) the short sides of the triangles will be 4 and 6.

$a^2 + b^2 = c^2$
$4^2 + 6^2 =$
$16 + 36 =$
$52 =$
$\sqrt{52} = c$
$c \approx 7.21$ units

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-3 - 3}{0 - -4}$$

$m \approx -1.5$

4. **Sheet 8** answers: Answers may vary depending on the value of the run and rise identified.

   a. Cloncurry (2.2, 6.3) to Hughenden (4.4, 6.3)

   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

   $d = \sqrt{(4.4 - 2.2)^2 + (6.3 - 6.3)^2}$

   $d = \sqrt{2.2^2}$

   $d = 2.2$

   Distance $= 2.2 \times 250$
   $= 550$ km

   b. Cunnamulla (5.7, 0.7) to Charleville (6, 2)

   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

   $d = \sqrt{(6 - 5.7)^2 + (2 - 0.7)^2}$

   $d = \sqrt{0.3^2 + 1.3^2}$

   $d = \sqrt{1.78}$

   $d \approx 1.33$

   Distance $\approx 1.33 \times 250$
   ≈ 332 km

   c. Emerald (7, 4.4) to Longreach (4.8, 4.4)

   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

   $d = \sqrt{(7.0 - 4.8)^2 + (4.4 - 4.4)^2}$

   $d = \sqrt{2.2^2}$

   $d = 2.2$

   Distance $= 2.2 \times 250$
   $= 550$ km

5. If students are unable to research distances, please supply them with the answers a, b and c:

   a. Cloncurry to Hughenden = 390 km in a straight line

   b. Cunnamulla to Charleville = 200 km in a straight line

   c. Emerald to Longreach = 420 km in a straight line

   d. The calculated distances are much larger than the actual distances.

   e. Issues with the sheet may include:

   - an incorrect scale or a scale that is rounded to an approximate value
   - a Cartesian grid that is slightly out
   - a Cartesian grid from which it is hard to estimate the values of coordinates that lie between the grid lines.

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Lesson 11

Topic: Linear and non-linear relationships

Calculating the midpoint of a line segment

Lesson concepts

- Representations — Linear models
- Functions — Graphs
- Length — Relationships
- Real numbers — Pythagoras

Lesson notes

Students will calculate the midpoint between two points.

Lesson answers

1. a. A = (0, 1); B = (4, 5)

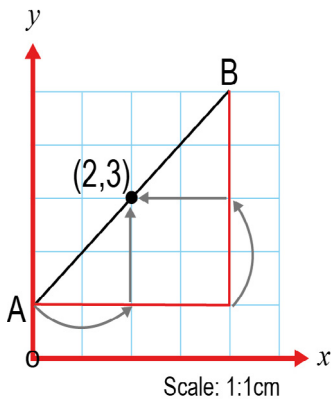
x midpoint $= \frac{0 + 4}{2}$

$= 2$

y midpoint $= \frac{1 + 5}{2}$

$= 3$

$\therefore$ midpoint = (2, 3)



b. A = (1, 1); B = (3, 4)

x midpoint $= \frac{1 + 3}{2}$

$= 2$

y midpoint $= \frac{1 + 4}{2}$

$= 2.5$

$\therefore$ midpoint = (2, 2.5)

2. a. Midpoint

Midpoint x value $= \frac{x_1 + x_2}{2}$

$= \frac{1 + 5}{2}$

$= \frac{6}{2}$

$= 3$

Midpoint y value $= \frac{y_1 + y_2}{2}$

$= \frac{2 + 4}{2}$

$= \frac{6}{2}$

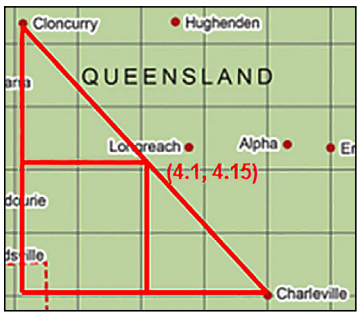
$= 3$

The midpoint is (3, 3).

b. The midpoint is (3.5, 3.5).

3. Due to the nature of this map, all answers given are approximate. Allow for a 0.2 coordinate error in your responses.

Midpoint of Charleville and Cloncurry



Coordinates for Cloncurry = (2.2, 6.3)

Coordinates for Charleville = (6, 2)

$$\text{Midpoint } x \text{ value} = \frac{x_1 + x_2}{2}$$
$$= \frac{2.2 + 6}{2}$$
$$= 4.1$$

$$\text{Midpoint } y \text{ value} = \frac{y_1 + y_2}{2}$$
$$= \frac{6.3 + 2}{2}$$
$$= 4.15$$

The midpoint is (4.1, 4.15).

Midpoint of Mackay and Brisbane

Mackay	(8.2, 6.1)
Brisbane	(10.4, 1.3)
The midpoint is	(9.3, 3.7).

Midpoint of Alpha and Rockhampton

Alpha	(6.3, 4.4)
Rockhampton	(8.9, 4.5)
The midpoint is	(7.6, 4.45).

4. Using the method as shown in Question 3:
Mt Isa (1.3, 6.5) Brisbane (10.4, 1.3)
The coordinates of the halfway point are (5.85, 3.9).

5. **Sheet 9** answers:

1.

Line 1: (–1,4) and (4,1)		Line 2: (–1,–4) and (4,–2)	
x value $= \frac{x_1 + x_2}{2}$ $= \frac{-1 + 4}{2}$ $= 1.5$	y value $= \frac{y_1 + y_2}{2}$ $= \frac{4 + 1}{2}$ $= 2.5$	x value $= \frac{x_1 + x_2}{2}$ $= \frac{-1 + 4}{2}$ $= 1.5$	y value $= \frac{y_1 + y_2}{2}$ $= \frac{-4 + -2}{2}$ $= -3$
Midpoint = (1.5, 2.5)		Midpoint = (1.5, –3)	

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Line 3: (–3 , 5) and (–1 , 1)		Line 4: (–4 , –2) and (1 , –1)	
x value $= \frac{x_1 + x_2}{2}$ $= \frac{-1 + -3}{2}$ $= -2$	y value $= \frac{y_1 + y_2}{2}$ $= \frac{1 + 5}{2}$ $= 3$	x value $= \frac{x_1 + x_2}{2}$ $= \frac{1 + -4}{2}$ $= -1.5$	y value $= \frac{y_1 + y_2}{2}$ $= \frac{-1 + -2}{2}$ $= -1.5$
Midpoint = (–2, 3)		Midpoint = (–1.5, –1.5)	

2.

Line a: (–5,1) and (5,–4)		Line b: (–4,–3) and (–1,6)	
x value $= \frac{x_1 + x_2}{2}$ $= \frac{-5 + 5}{2}$ $= 0$	y value $= \frac{y_1 + y_2}{2}$ $= \frac{1 + -4}{2}$ $= -1.5$	x value $= \frac{x_1 + x_2}{2}$ $= \frac{-4 + -1}{2}$ $= -2.5$	y value $= \frac{y_1 + y_2}{2}$ $= \frac{-3 + 6}{2}$ $= 1.5$
Midpoint = (0, –1.5)		Midpoint = (–2.5, 1.5)	

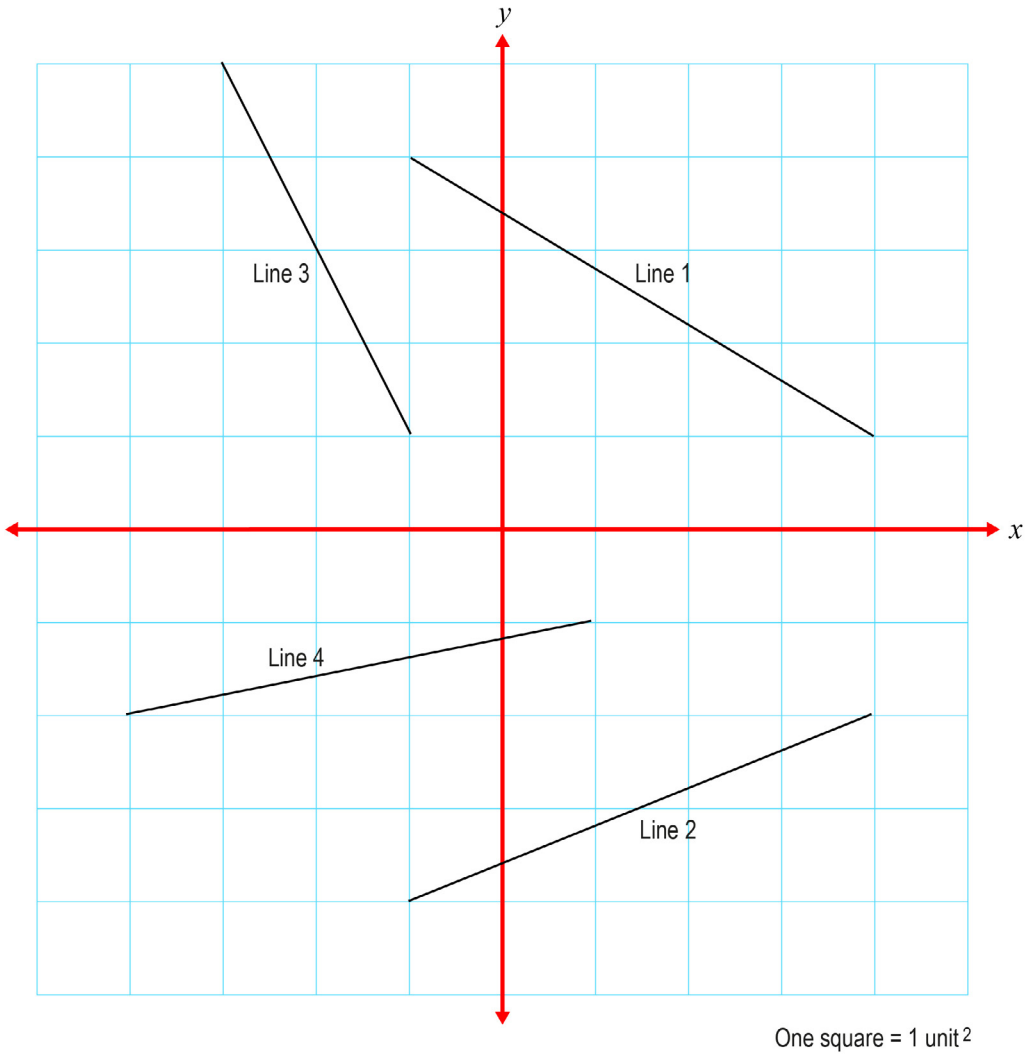
Line c: (1, 3) and (–1, 0)	
x value $= \frac{x_1 + x_2}{2}$ $= \frac{1 + -1}{2}$ $= 0$	y value $= \frac{y_1 + y_2}{2}$ $= \frac{3 + 0}{2}$ $= 1.5$
Midpoint = (0, 1.5)	

3.

Endpoint: (–2, 9) and (x_2, y_2) Midpoint: (1, –4)	
$1 = \frac{-2 + x_2}{2}$ $1 \times 2 = x_2 + -2$ $2 - -2 = x_2$ $4 = x_2$	$-4 = \frac{9 + y_2}{2}$ $-4 \times 2 = 9 + y_2$ $-8 - 9 = y_2$ $-17 = y_2$
Second endpoint = (4, 1)	

How steep is it?

Sheet 7 Answers



1. Consider the lines on the axes above. Without calculating:

   a. Identify the lines that have positive gradients. Explain how you know this.

   Lines 2 and 4 have positive gradients. They rise from left to right.

   b. Identify the lines that have negative gradients. Explain how you know this.

   Lines 1 and 3 have negative gradients. They fall from left to right.

   c. List the order of 'steepness' of the lines from least steep to most steep.

   Answers may vary depending on how well you can estimate the steepness of a slope.
   The correct order is: Line 4, Line 2, Line 1, Line 3.

Sheet 7 Answers

2. Calculate the gradient of each line. Show clearly how you are obtaining your solutions.

Line 1: $m = \frac{\text{rise}}{\text{run}}$
$= -\frac{3}{5}$
$= -0.6$

Line 2: $m = \frac{\text{rise}}{\text{run}}$
$= \frac{2}{5}$
$= 0.4$

Line 3: $m = \frac{\text{rise}}{\text{run}}$
$= -\frac{4}{2}$
$= -2$

Line 4: $m = \frac{\text{rise}}{\text{run}}$
$= \frac{1}{5}$
$= 0.2$

3. Consider the values you obtained for the gradients of each line in Question 2. Does this correspond to your initial answers for Question 1c?

Answers will vary depending on how well you estimated the steepness.

4. Use the formula, $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$ to find the gradient of the lines drawn between the following points:

a. (–5, 1) and (5, –4)

$\text{gradient} = \frac{\text{rise}}{\text{run}}$
$= \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-4 - 1}{5 - -5}$
$= \frac{-5}{10}$
$= -0.5$

b. (–4, –3) and (–1, 6)

$\text{gradient} = \frac{\text{rise}}{\text{run}}$
$= \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{6 - -3}{-1 - -4}$
$= \frac{9}{3}$
$= 3$

c. (1, 3) and (–1, 0)

$\text{gradient} = \frac{\text{rise}}{\text{run}}$
$= \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{0 - 3}{-1 - 1}$
$= \frac{-3}{-2}$
$= 1.5$

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