MATHS

Lesson 8

Year 9

Topic: Linear and non-linear relationships

Calculating gradients 2

Today you will:

- reinforce understanding of calculating gradients.

Resources

Digital

Video — Understanding the c value in a linear equation (2:53)

Find and prepare

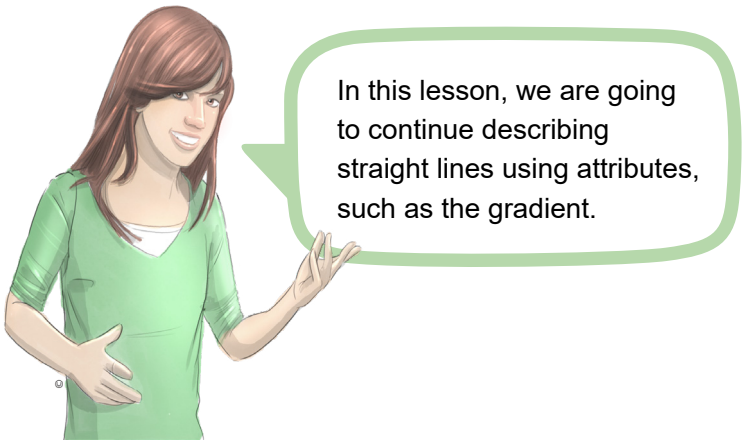
Sheet 7 — How steep is it?

Key terms

For definitions and explanations of terms, please see the Glossary.

Lesson

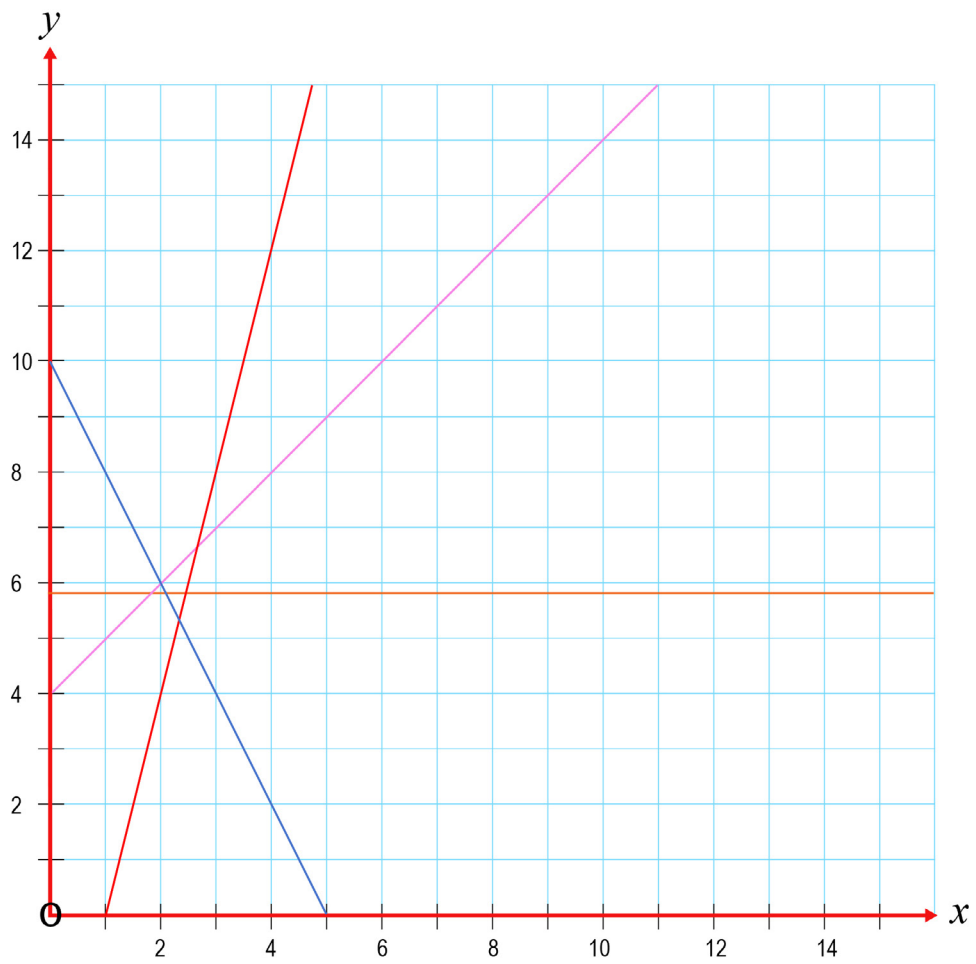
Different ways of describing and calculating the gradient



Remember the mathematical symbol for gradient is m.

1. Match the lines on the following graph with the appropriate value of m. Write the colour of the line under the corresponding m value.

$m = 1$	$m = -2$	$m = 4$	$m = 0$



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Explore the effects of manipulating the gradient (m) and constant (c) in the general linear equation ($y = mx + c$)

2. Explore the link between gradient, y-intercept and features on a linear graph (straight line shown). Work through the following steps and answer the questions.

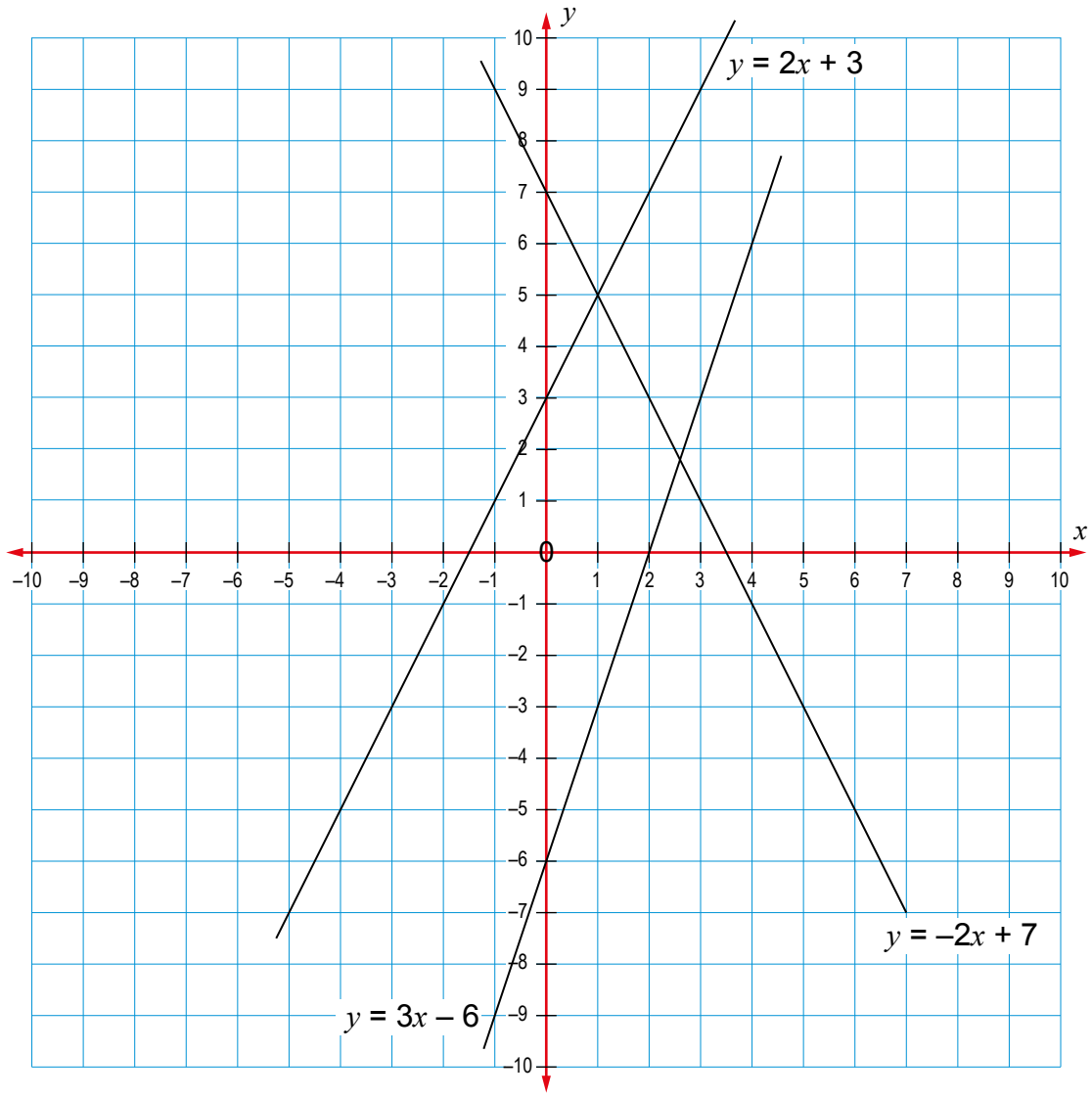
   a. State the gradient for the line $y = 2x + 3$.

   b. Where does the line $y = 2x + 3$ cut the y-axis (that is, the y-intercept)?

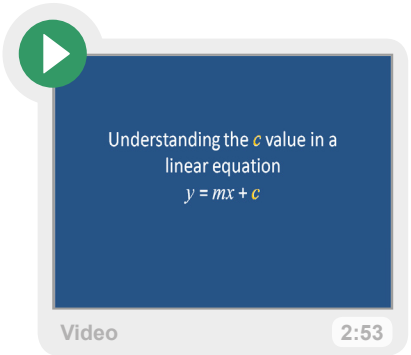
   c. Identify where in the equation of the line $y = 2x + 3$ you find the gradient and y-intercept (your answers to a and b).

   d. Repeat steps a, b and c for the line $y = -2x + 7$.

   e. Repeat steps a, b and c for the line $y = 3x - 6$.



3. Watch the **Video — Understanding the c value in a linear equation**.



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In summary: Parts of a linear graph

- **Gradient** (also called **slope**) is a measure of steepness.
- The greater the value of gradient, the steeper the line.
- A **positive** gradient *rises* left to right (↗).
- A **negative** gradient *falls* left to right (↘).
- A gradient that is **zero** is a *horizontal line* (→).
- An **undefined** gradient is a *vertical line* (↑).

Rise: the vertical distance between two points on the graph.

Run: the horizontal distance between those same two points on the graph.

***y*-intercept:** the point where the line intercepts the y-axis.

Generalisations from your investigation

Consider the function $y = mx + c$ which describes *linear* graphs

- m determines *gradient* ($\frac{\text{rise}}{\text{run}}$)
- c determines *y-intercept*.

Example

Equation: $y = 1x + 4$

$m = 1$

$c = 4$

This means when the equation is graphed, the:

- y-intercept will intercept the y-axis at 4
- the gradient will be 1.

When the value of m is changed, the gradient (slope) of the plot is affected.

When the value of c is changed, the point where the line passes through the y-axis (y-intercept) is affected.

4. Find the gradient (m) and the y-intercept (c) of the lines on the following graph. The first one has been done for you. Finish by using your answers to write the linear function for each line. Complete your working in your Maths notebook.

Example: Line 1

$m = \frac{\text{rise}}{\text{run}}$

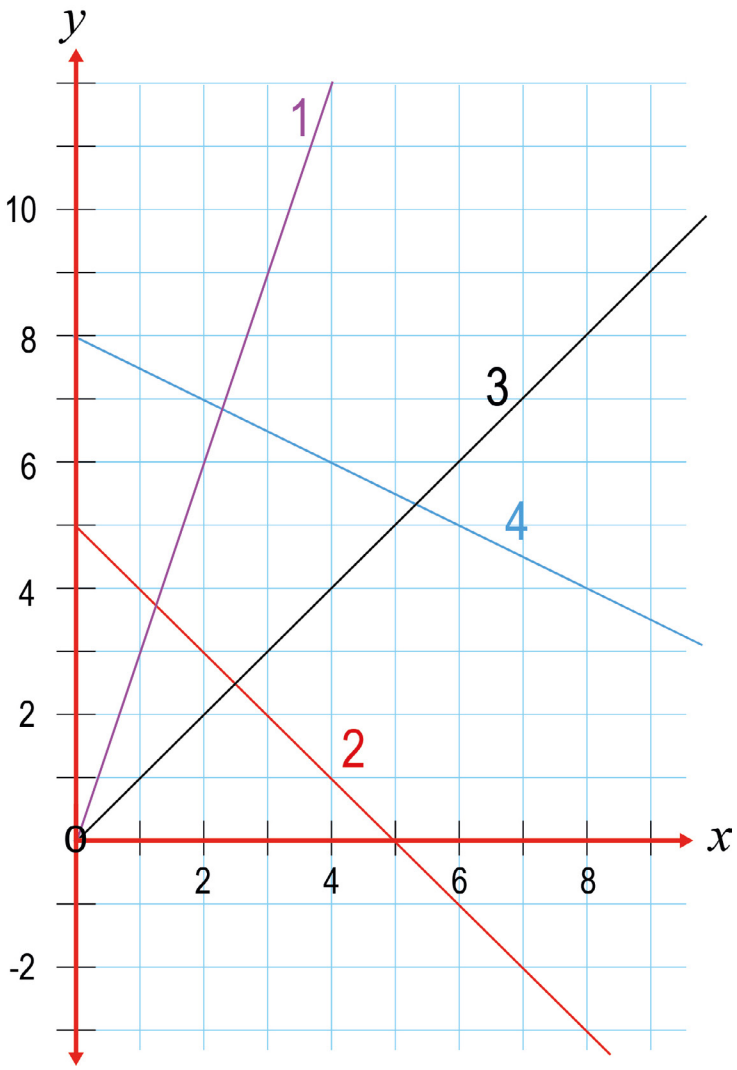
$= \frac{3}{1}$

$= 3$

The gradient is 3.

y-intercept = 0

Equation: $y = 3x$



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Explore the formal rule for calculating gradient

Ways to calculate gradient

Method 1

1. Establish the x- and y-axes.
2. Establish the scales and the coordinates for each end point of the line (for this example, one square represents 1 unit²).
3. Use these coordinates to establish the gradient of the line by finding the rise (change in y values) and the run (change in x values).

Rise = 3 – 1

= 2

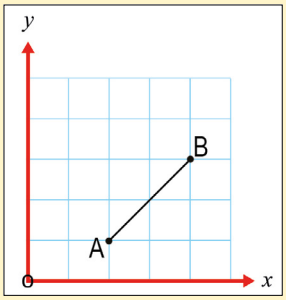
Run = 4 – 2

= 2

gradient = $\frac{\text{rise}}{\text{run}}$

= $\frac{2}{2}$

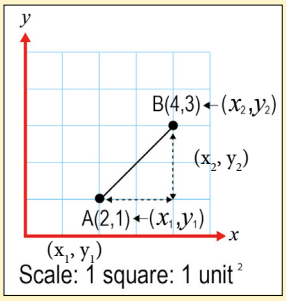
= 1



Ways to calculate gradient

Method 2

Let A be the first point. Then, the x-coordinate of A can be written as x_1 and the y-coordinate can be labelled y_1. Similarly, let B be the second point so its coordinates are (x_2, y_2). Now we can use these coordinates to write a rule to calculate rise and run.



Rise = $y_2 - y_1$

Run = $x_2 - x_1$

gradient = $\frac{\text{rise}}{\text{run}}$

= $\frac{y_2 - y_1}{x_2 - x_1}$

Coordinates of A are (2, 1) and B are (4, 3).

gradient = $\frac{y_2 - y_1}{x_2 - x_1}$

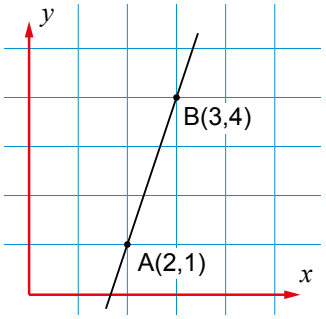
= $\frac{3 - 1}{4 - 2}$

= 1

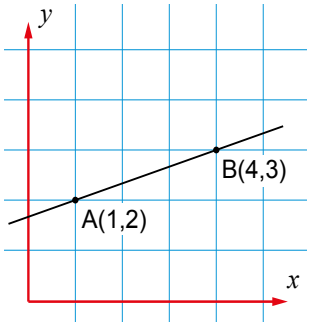
5. For the following graphs:
   - state whether the lines have a negative or positive gradient
   - calculate the gradient of each line (using the rule from Method 2 demonstrated previously).

   Hint: If you find Method 2 difficult, use Method 1.

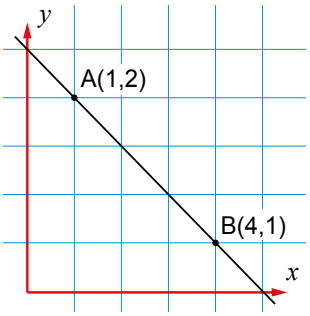
a.



b.



c.



6. Find the gradient of the line drawn between the points (−2, 1) and (−2, 5). What type of line is it?
7. Calculate the gradients of the lines in **Sheet 7 —** How steep is it?
8. Consider gradients in a real-life context.

   a. Calculate the gradient of a staircase that has steps that are 20 cm wide (run) and the gap (rise) between each step is also 20 cm.

   b. If you increase the width of the step to 40 cm, what will happen to the gradient? How will the look of the staircase change?

   c. The industry standard for wheelchair access is that a ramp must have a gradient of 1:10. This means the gradient must be 0.1 (1 ÷ 10) or less to be legal. Determine whether the ramp in the following image is legal.

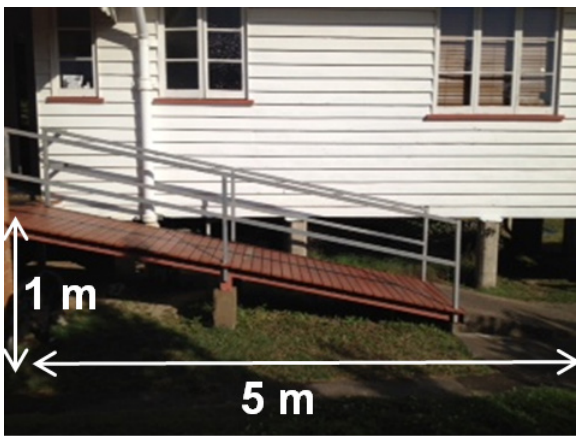


Image courtesy of Kim McClelland. Used with permission

   d. How long would the run of the ramp need to be to make it suitable for wheelchair access?

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Gradients can be identified all around us and can help us stay safe. For example, road signs use gradients to alert drivers of a steep road ahead and they identify the gradient as a percentage. This sign indicates the road has a gradient of 10% That is a gradient of 0.1. What would a gradient of 1 look like and how easy or safe do you think it would be to drive up or down a road this steep?

