MATHS

Lesson 9

Year 9

Topic: Linear and non-linear relationships

Calculating the distance between two points 1

Today you will:

- use Pythagoras' theorem to calculate the hypotenuse.

Resources

Digital

Video — Pythagoras' theorem (4:45)

Find and prepare

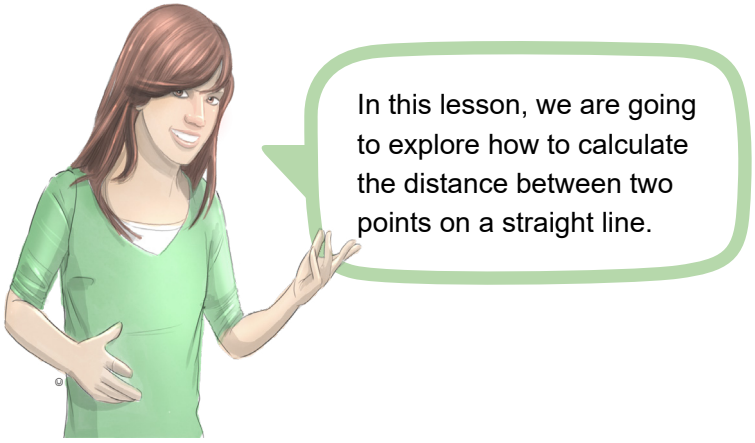
Maths notebook

Key terms

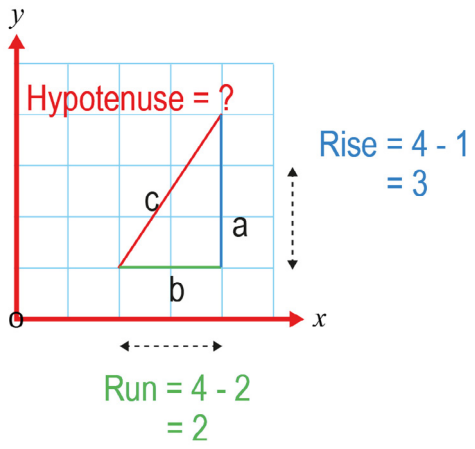
For definitions and explanations of terms, please see the Glossary.

Lesson

Relate gradient to a right-angled triangle



This diagram shows the rise and run of the line segment. By drawing in the vertical and horizontal lines that show rise and run, we make a right-angled triangle. The oblique line (the red one) on the triangle is called the hypotenuse. The hypotenuse is the longest side of a triangle and is always opposite the right angle.

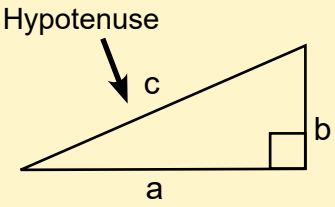


Hypotenuse

There are two simple ways to find the hypotenuse of a right-angled triangle:

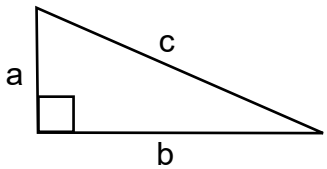
- it is always the side opposite the right-angle
- it is always the longest side of a right-angled triangle.

In this example, the hypotenuse is side c.

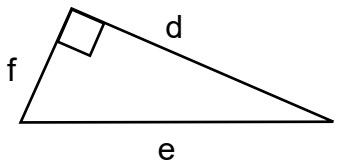


1. Identify the hypotenuse on these triangles.

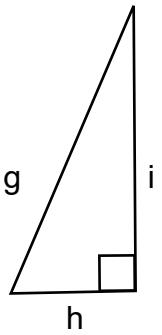
a.



b.



c.



2. Use the following geoboard to help compare the length of the hypotenuse of a right-angled triangle to the length of the other two sides. Work through the following steps and answer the questions.

   a. Draw a right-angled triangle on the geoboard. Use a base length of 3 cm and a perpendicular height of 4 cm for the triangle.

   b. Identify the hypotenuse on the right-angled triangle.

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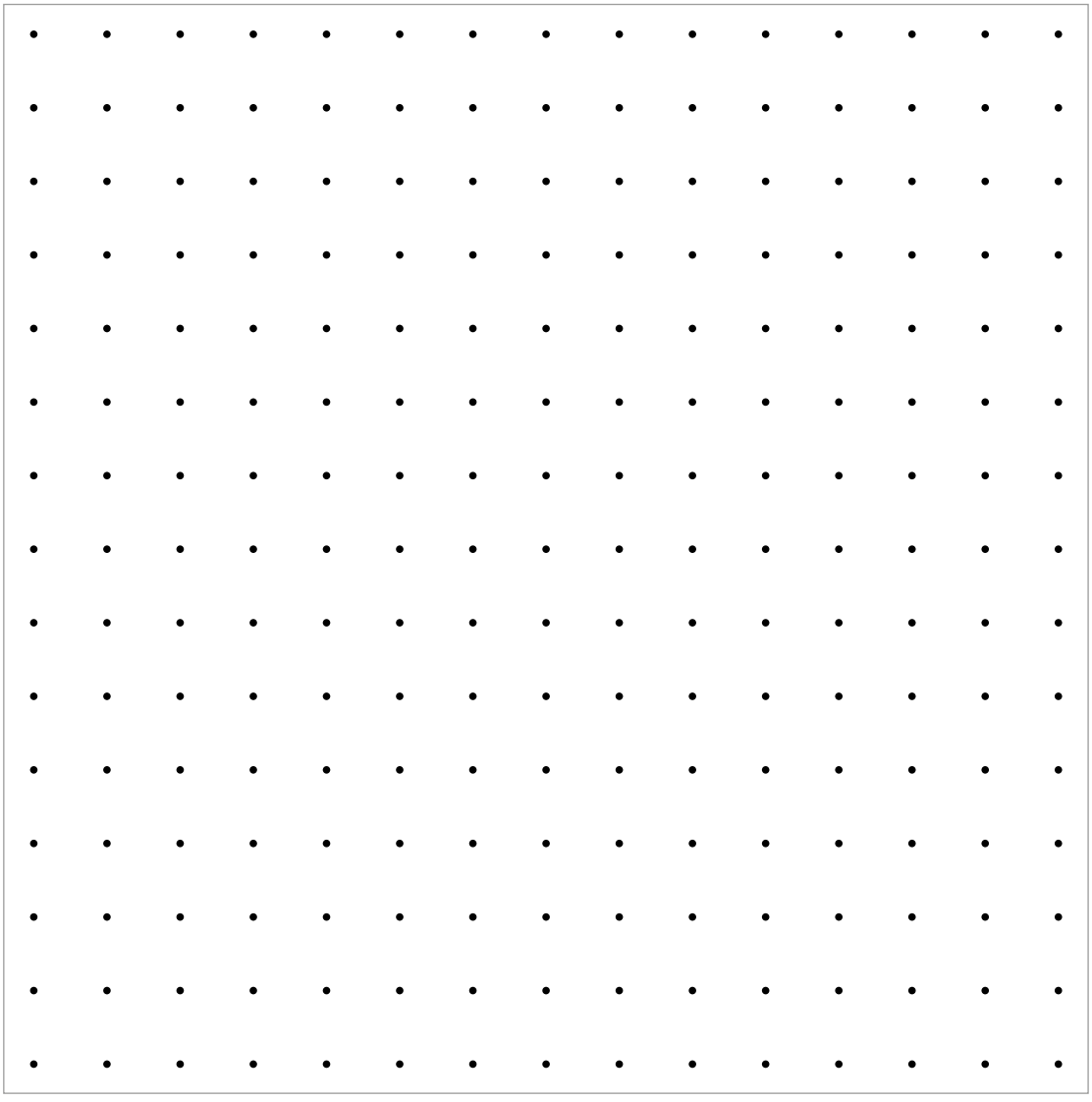
c. Estimate the length of the hypotenuse.

d. Measure the length of the hypotenuse using a ruler.

e. How did your estimate compare to your measurement?

f. How does the length of the hypotenuse compare to the base?

g. How does the length of the hypotenuse compare to the perpendicular height?



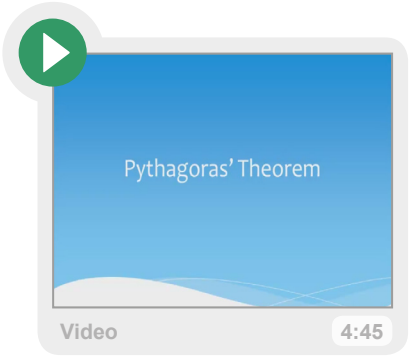
3. Repeat the steps above using a right-angled triangle with a base of 12 cm and a perpendicular height of 5 cm.

Use Pythagoras' theorem to calculate the hypotenuse



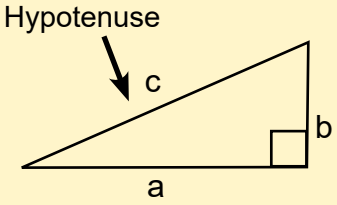
The theorem is named after the Greek mathematician Pythagoras, who proved that if you make a square on each of the three sides of a right-angled triangle the area of the square whose side is attached to the hypotenuse is equal to the sum of the area of the squares attached to the other two sides.

4. Watch the **Video — Pythagoras' theorem**.



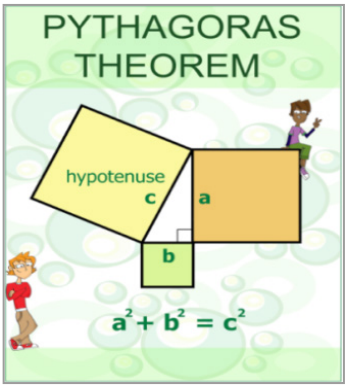
Pythagoras' theorem

For a right-angled triangle, the square of the **hypotenuse** of a right-angled triangle equals the sum of the squares of the lengths of the other two sides. This can be written using the following rule:



$$a^2 + b^2 = c^2$$

Note: Sometimes the formula is rearranged and referred to as $c^2 = a^2 + b^2$.



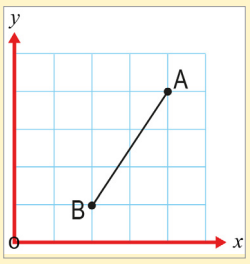


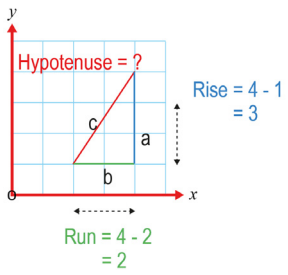
We will now use this theorem to calculate the distance between two points on a line.

Calculating the distance between two points

Example:

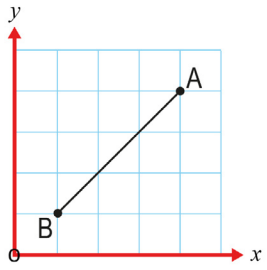
Find the distance between points A and B.



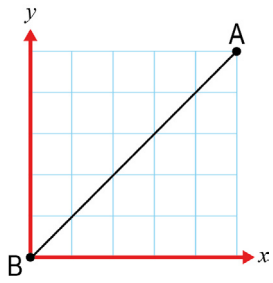
Think	Write
Draw a line to make a right-angled triangle. Label the sides a, b and c (hypotenuse). Calculate the length of the perpendicular sides (a and b).	
Write Pythagoras' theorem.	$a^2 + b^2 = c^2$
Substitute values for a and b, into the formula. Calculate the unknown. Write the answer to two decimal places.	$3^2 + 2^2 = c^2$ $9 + 4 = c^2$ $13 = c^2$ $\sqrt{13} = c$ $3.61 \approx c$
Write a statement to communicate the final solution.	The length of the line is 3.61 units.

5. Find the distance between the two points for the 2 line segments shown.

a.



b.



Another example:

Construct a right-angled triangle on the **geoboard** with sides of 3 cm and 4 cm and find the length of the hypotenuse. Identify the hypotenuse (c) and the two perpendicular sides (a and b). That is, a = 3 cm, b = 4 cm, c = ? or a = 4 cm, b = 3 cm, c = ?

Use Pythagoras' theorem to calculate the hypotenuse (the unknown).

Think	**Write**
Write the rule	$c^2 = a^2 + b^2$
Substitute known values	$= 3^2 + 4^2$
Calculate c^2	$= 9 + 16$
	$= 25$
Calculate c	$c = \sqrt{25}$
State the solution	Hypotenuse is 5 cm

You can reduce the steps you write while still communicating your thinking, for example: a = 5 cm, b = 7 cm, c = ?

Think	**Write**
Write the rule	$c^2 = a^2 + b^2$
Substitute known values	$= 5^2 + 7^2$
Calculate c	$c = \sqrt{74}$
State the solution	Hypotenuse is 8.6 cm

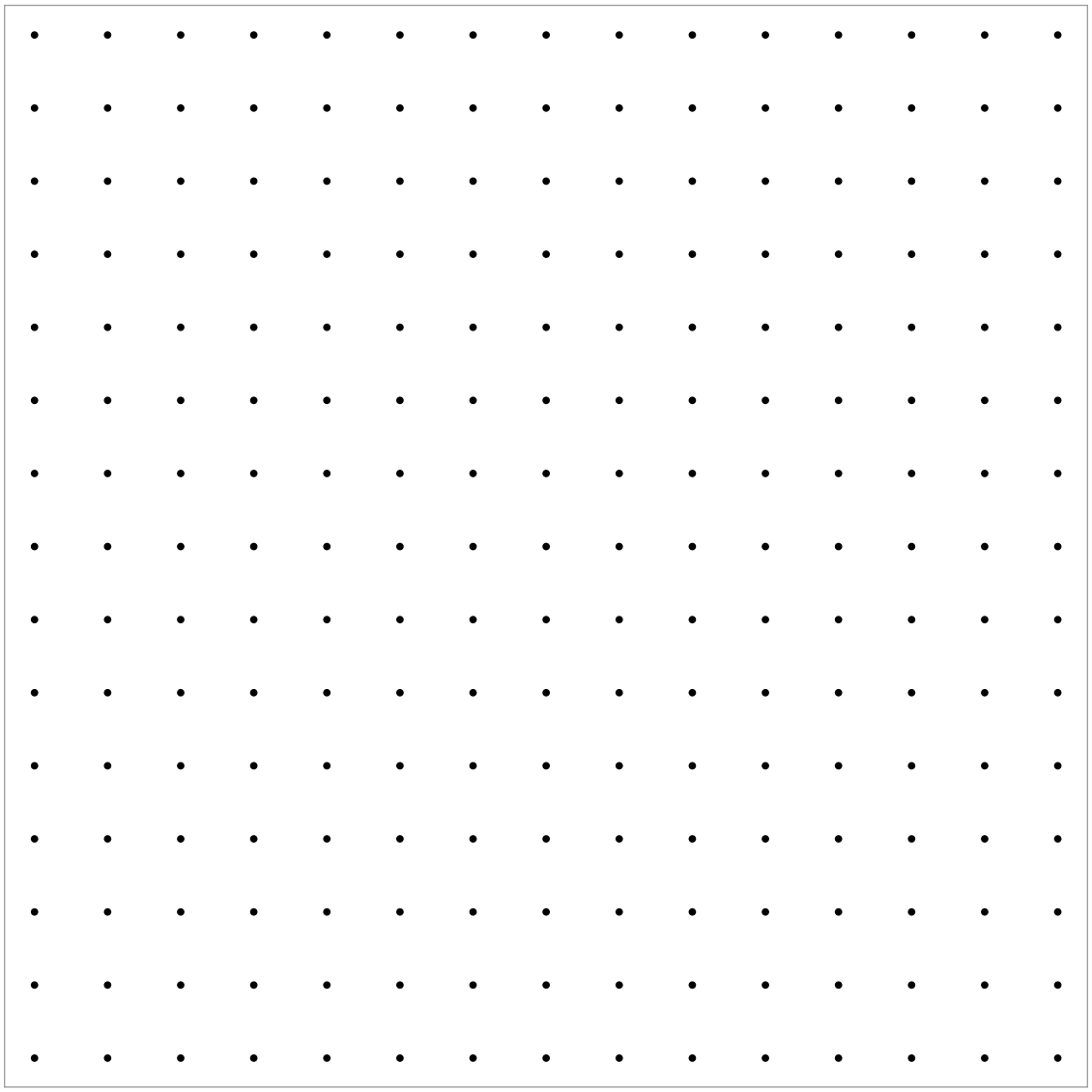
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6. Create three triangles on the following **geoboard** using the measurements shown. Use Pythagoras' theorem (show working in your Maths notebook) to calculate the length of the hypotenuse.

   a. Rise = 4 units and Run = 3 units

   b. Rise = 6 units and Run = 8 units

   c. Rise = 5 units and Run = 7 units



Hot tip

Always put units in your answer. If none are given, write the word 'units' after the number.



In this lesson, we applied our understanding of slope to right-angled triangles. It seems that we just keep coming back to right-angled triangles! That's because these triangles are one of the most basic shapes, and other shapes can usually be broken down into right-angled triangles. This means the things we learn about these triangles can be used in lots of other places too.